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Gauge invariance, gauge-parameter independence and properties of Green functions†

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Abstract:
The application of the background-field method to the electroweak Standard Model is reviewed and further explored. Special emphasis is put on questions of gauge invariance and gauge-parameter (in-)dependence. Owing to the gauge invariance of the background-field effective action, the vertex functions obey simple Ward identities which imply important properties of the vertex functions. Carrying out the renormalization in a way respecting background-field gauge invariance leads to considerable simplifications. The generalization of the background-field method to the non-linear realization of the scalar sector of the Standard Model is illustrated. Furthermore, the interplay between gauge independence of the S-matrix and Ward identities of vertex functions is investigated. Finally, the Standard Model contributions to the S, T, and U parameters are calculated and discussed within the background-field method.

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GAUGE INVARIANCE, GAUGE-PARAMETER INDEPENDENCE AND PROPERTIES OF GREEN FUNCTIONS

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ABSTRACT

The application of the background-field method to the electroweak Standard Model is reviewed and further explored. Special emphasis is put on questions of gauge invariance and gauge-parameter (in-)dependence. Owing to the gauge invariance of the background-field effective action, the vertex functions obey simple Ward identities which imply important properties of the vertex functions. Carrying out the renormalization in a way respecting background-field gauge invariance leads to considerable simplifications. The generalization of the background-field method to the non-linear realization of the scalar sector of the Standard Model is illustrated. Furthermore, the interplay between gauge independence of the S-matrix and Ward identities of vertex functions is investigated. Finally, the Standard Model contributions to the S, T, and U parameters are calculated and discussed within the background-field method.

1. Introduction

The properties of gauge invariance and gauge-parameter independence, which are inherent in all kinds of gauge theories, have recently gained renewed interest. The question of gauge dependence arises automatically whenever physical observables, i.e. S-matrix elements, are not strictly calculated order by order in perturbation theory. However, mixing different orders of the perturbative expansion is sometimes unavoidable. For example, the introduction of finite-width effects for unstable particles or of running couplings can only be achieved by a resummation of certain subsets of Feynman diagrams. Moreover, single off-shell vertex functions have been parametrized by so-called form factors in the literature. The physical significance of such objects is always questionable. Every definition of quantities from incomplete parts of S-matrix elements (in a fixed order of perturbation theory) is necessarily based on conventions but not on physical grounds.

At this point a few remarks on the difference between gauge invariance and gauge-parameter independence are in order. Strictly speaking, one can call only such objects

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gauge-invariant which are singlets with respect to gauge transformations. However, the gauge invariance of the underlying Lagrangian has to be broken in order to quantize the fields in perturbation theory. To this end a gauge-fixing term is added to the Lagrangian depending on one or more free (gauge-)parameters, which drop out in complete S-matrix elements. If a quantity depends on the gauge parameters, it depends on the gauge-fixing procedure. On the other hand, it can be shown that Green functions of gauge-invariant operators are independent of the method of gauge fixing and thus gauge-parameter-independent. However, the inverse conclusion is wrong in general: gauge-parameter independence does *not* necessarily indicate gauge invariance.

The fact that the gauge-parameter dependence of individual vertex functions (self-energies, vertex corrections, etc.) is compensated within complete S-matrix elements motivated several authors to rearrange the gauge-dependent parts between different vertex functions resulting in definitions of separately gauge-parameter-independent building blocks. Since such splittings of vertex functions are not uniquely determined, different proposals were made in the literature. For example, in the context of four-fermion processes different running couplings [1,2] have been defined. A more general procedure for eliminating the gauge-parameter-dependent parts of vertex functions is given by the so-called pinch technique (PT) [3,4,5,6]. All these approaches have the common aim to define gauge-parameter-independent "vertex functions" with improved theoretical properties. In this context it should be noticed that these procedures are not free of problems. The methods of Refs. 1,2 have no natural generalization beyond four-fermion processes. On the other hand, the application of the PT algorithm is not always clear, and the universality (process independence) of the PT "vertex functions" has not rigorously been proven but only been verified from examples [5,6].

Therefore, we pursue a completely different approach and study directly consequences of the underlying gauge invariance for vertex functions. The background-field method (BFM) [7,8,9] represents a well-suited framework for such investigations. In the BFM the effective action, which generates the vertex functions, is manifestly gauge-invariant, and this invariance implies simple Ward identities for vertex functions. For the calculation of S-matrix elements, tree-like structures are formed with these vertex functions, where the gauge fixing of the genuine tree part can be chosen arbitrarily.

In Refs. 10,11 the BFM was applied to the electroweak Standard Model (SM), and the consequences of the Ward identities were discussed. The renormalization[a] was carried out in a gauge-invariant way, which led to considerable simplifications. Moreover, it was shown that the Ward identities imply several improved properties for vertex functions (compared to the conventional formalism) concerning ultraviolet, infrared or high-energy behavior. Furthermore, actual loop calculations of S-matrix elements in general become simpler using the BFM. This is mainly due to the freedom of choosing the gauge for the tree parts independently from the one in the loops.

[a]The renormalization of the electroweak SM without fermions was also discussed in Ref. 12.

The BFM brings together two important features: the gauge invariance of the effective action and the clear distinction between classical and quantum parts of fields. This fact renders the BFM well-suited for integrating out heavy fields at one-loop level directly in the path integral. Firstly, the tree-level and one-loop effects can be isolated in the path integral very easily. Secondly, choosing a definite gauge (e.g. the unitary gauge) for the background fields drastically simplifies intermediate steps in the $1/M$-expansion for the heavy field of mass M. Thus, a one-loop effective Lagrangian can be directly derived and by inverting the transformation to the definite background gauge after the $1/M$-expansion one recovers a manifestly gauge-invariant result. Such a procedure was worked out in Ref. 13 and applied to an $\mathrm{SU(2)_W}$ gauge theory and the SM.

In Ref. 10 it was realized that the building blocks obtained within the PT in QCD[b] and the SM coincide with the corresponding BFM vertex functions in 't Hooft–Feynman gauge. This observation and further investigations in Ref. 10 clarified the origin of certain desirable properties [4] noticed for the PT "vertex functions". In particular, in the BFM the QED-like Ward identities are derived from gauge invariance and imply all the other properties as shown in Ref. 10. Since this is true in the BFM for arbitrary gauges of the quantum fields, the above-mentioned desirable properties are related to the gauge invariance of the effective action rather than to the absence of gauge-parameter dependence.

In this article we first review the basic results of the application of the BFM to the SM. They have been worked out in Ref. 11 for the usual linear realization of the scalar Higgs doublet. The generalization of the BFM to the non-linear realization of the scalar sector was described in Ref. 13. We further explore the connection between gauge-parameter-independent formulations and the BFM. Finally, we focus on the SM contributions to the S, T, and U parameters, which have been originally introduced [15] in order to quantify new-physics effects beyond the SM entering via vacuum polarization only. Comparing the BFM results for the S, T, and U parameters with the ones obtained within the PT [16], the relevance of the latter is discussed.

The article is organized as follows: in section 2 we review the application of the background field method to the electroweak SM. The renormalization of the SM in the BFM is discussed in section 3. In section 4 we summarize the virtues of the BFM in the formulation of the SM with a non-linear realization of the Higgs sector. In section 5 we elaborate on the connection between gauge-parameter independence of vertex functions and symmetry relations. As a further illustration we treat the S, T, and U parameters in the BFM in section 6.

[b] In QCD this fact was also pointed out in Ref. 14.

2. The Background-Field Method for the Electroweak Standard Model

2.1. The Construction of the gauge-invariant Effective Action

The background-field method [7,8] (BFM) is a technique for quantizing gauge theories without losing explicit gauge invariance of the effective action. This is done by decomposing the usual fields $\hat{\varphi}$ in the classical Lagrangian $\mathcal{L}_{\mathrm{C}}$ into background fields $\hat{\varphi}$ and quantum fields φ,

$$\mathcal{L}_{\mathrm{C}}(\hat{\varphi}) \to \mathcal{L}_{\mathrm{C}}(\hat{\varphi}+\varphi). \tag{1}$$

While the background fields are treated as external sources, only the quantum fields are variables of integration in the functional integral. A gauge-fixing term is added which breaks only the invariance with respect to quantum gauge transformations but retains the invariance of the functional integral with respect to background-field gauge transformations. From the functional integral an effective action $\Gamma[\hat{\varphi}]$ for the background fields is derived which is invariant under gauge transformations of the background fields and thus gauge-invariant.

The S-matrix is constructed by forming trees with vertex functions from $\Gamma[\hat{\varphi}]$ joined by background-field propagators. These propagators are defined by adding a gauge-fixing term to $\Gamma[\hat{\varphi}]$. This gauge-fixing term is only relevant for the construction of connected Green functions and S-matrix elements. It is not related to the term used to fix the gauge inside loop diagrams, i.e. in the functional integral, and the associated gauge parameters ξ_B^i only enter tree level quantities but not the higher-order contributions to the vertex functions. In particular, in linear background gauges only the tree-level propagators are affected by the background gauge fixing. The equivalence of the S-matrix in the BFM to the conventional one has been proven in Refs. 9,17,18.

For our discussion of the SM we use the conventions of Refs. 11,19. The complex scalar $\mathrm{SU(2)_W}$ doublet field of the minimal Higgs sector is written as the sum of a background Higgs field $\hat{\Phi}$ having the usual non-vanishing vacuum expectation value v, and a quantum Higgs field Φ whose vacuum expectation value is zero:

$$\hat{\Phi}(x) = \begin{pmatrix} \hat{\phi}^+(x) \\ \frac{1}{\sqrt{2}}(v+\hat{H}(x)+i\hat{\chi}(x)) \end{pmatrix}, \qquad \Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(H(x)+i\chi(x)) \end{pmatrix}. \tag{2}$$

Here $\hat{H}$ and H denote the physical background and quantum Higgs field, respectively, and $\hat{\phi}^+, \hat{\chi}, \phi^+, \chi$ are the unphysical Goldstone fields.

The generalization of the 't Hooft gauge fixing to the BFM [20] reads

$$\mathcal{L}_{\mathrm{GF}} = -\frac{1}{2\xi_Q^W}\Big[(\delta^{ac}\partial_\mu + g_2\varepsilon^{abc}\hat{W}^b_\mu)W^{c,\mu} - ig_2\xi_Q^W\frac{1}{2}(\hat{\Phi}^\dagger_i\sigma^a_{ij}\Phi_j - \Phi^\dagger_i\sigma^a_{ij}\hat{\Phi}_j)\Big]^2$$

$$- \frac{1}{2\xi_Q^B}\Big[\partial_\mu B^\mu + ig_1 \xi_Q^B \frac{1}{2}(\hat{\Phi}_i^\dagger \Phi_i - \Phi_i^\dagger \hat{\Phi}_i)\Big]^2, \tag{3}$$

where $\hat{W}_\mu^a$, a=1,2,3, represents the triplet of gauge fields associated with the weak isospin group $\mathrm{SU(2)_W}$, and $\hat{B}_\mu$ the gauge field associated with the group $\mathrm{U(1)_Y}$ of weak hypercharge Y_W. The Pauli matrices are denoted by σ^a, $a = 1, 2, 3$, and ξ_Q^W, ξ_Q^B are parameters associated with the gauge fixing of the quantum fields, one for $\mathrm{SU(2)_W}$ and one for $\mathrm{U(1)_Y}$. In order to avoid tree-level mixing between the quantum A and Z fields, we set $\xi_Q = \xi_Q^W = \xi_Q^B$ in the following. Background-field gauge invariance implies that the background gauge fields appear only within a covariant derivative in the gauge-fixing term and that the terms in brackets transform according to the adjoint representation of the gauge group. The gauge-fixing term (Eq. (3)) translates to the conventional one upon replacing the background Higgs field by its vacuum expectation value and omitting the background $\mathrm{SU(2)_W}$ triplet field $\hat{W}_\mu^a$.

The vertex functions can be calculated directly from Feynman rules that distinguish between quantum and background fields. Whereas the quantum fields appear only inside loops, the background fields are associated with external lines. Apart from doubling of the gauge and Higgs fields, the BFM Feynman rules differ from the conventional ones only owing to the gauge-fixing and ghost terms. Because these terms are quadratic in the quantum fields, they affect only vertices that involve exactly two quantum fields and additional background fields. Since the gauge-fixing term is non-linear in the fields, the gauge parameter enters also the gauge-boson vertices. The fermion fields are treated as usual, they have the conventional Feynman rules, and no distinction needs to be made between external and internal fields. A complete set of BFM Feynman rules for the electroweak SM has been given in Ref. 11.

Despite the distinction between background and quantum fields, calculations in the BFM become in general simpler than in the conventional formalism. This is in particular the case in the 't Hooft–Feynman gauge ($\xi_Q = 1$) for the quantum fields where many vertices simplify a lot. Moreover, the gauge fixing of the background fields is totally unrelated to the gauge fixing of the quantum fields[17]. This freedom can be used to choose a particularly suitable background gauge, e.g. the unitary gauge. In this way the number of Feynman diagrams can considerably be reduced.

2.2. Ward Identities

As can be directly read off from Eqs. (21), (22) of Ref. 11, the invariance of the effective action under the background gauge transformations yields

$$\begin{aligned} 0 = \frac{\delta\Gamma}{\delta\hat{\theta}^A} = &-\partial_\mu \frac{\delta\Gamma}{\delta\hat{A}_\mu} - ie\Big(\hat{W}_\mu^+ \frac{\delta\Gamma}{\delta\hat{W}_\mu^+} - \hat{W}_\mu^- \frac{\delta\Gamma}{\delta\hat{W}_\mu^-}\Big) - ie\Big(\hat{\phi}^+ \frac{\delta\Gamma}{\delta\hat{\phi}^+} - \hat{\phi}^- \frac{\delta\Gamma}{\delta\hat{\phi}^-}\Big) \\ &+ ie\sum_f Q_f\Big(\bar{f}\frac{\delta\Gamma}{\delta\bar{f}} + \frac{\delta\Gamma}{\delta f} f\Big), \end{aligned} \tag{4}$$

$$
\begin{aligned}
0 = \frac{\delta\Gamma}{\delta\hat{\theta}^Z} = & -\partial_\mu \frac{\delta\Gamma}{\delta\hat{Z}_\mu} + ie\frac{c_W}{s_W}\Big(\hat{W}^+_\mu \frac{\delta\Gamma}{\delta\hat{W}^+_\mu} - \hat{W}^-_\mu \frac{\delta\Gamma}{\delta\hat{W}^-_\mu}\Big) \\
& + ie\frac{c_W^2 - s_W^2}{2c_W s_W}\Big(\hat{\phi}^+ \frac{\delta\Gamma}{\delta\hat{\phi}^+} - \hat{\phi}^- \frac{\delta\Gamma}{\delta\hat{\phi}^-}\Big) - e\frac{1}{2c_W s_W}\Big((v+\hat{H})\frac{\delta\Gamma}{\delta\hat{\chi}} - \hat{\chi}\frac{\delta\Gamma}{\delta\hat{H}}\Big) \\
& - ie\sum_f \Big(\bar{f}(v_f + a_f\gamma_5)\frac{\delta\Gamma}{\delta\bar{f}} + \frac{\delta\Gamma}{\delta f}(v_f - a_f\gamma_5) f\Big),
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
0 = \frac{\delta\Gamma}{\delta\hat{\theta}^\pm} = & -\partial_\mu \frac{\delta\Gamma}{\delta\hat{W}^\pm_\mu} \mp ie\hat{W}^\mp_\mu \Big(\frac{\delta\Gamma}{\delta\hat{A}_\mu} - \frac{c_W}{s_W}\frac{\delta\Gamma}{\delta\hat{Z}_\mu}\Big) \pm ie(\hat{A}_\mu - \frac{c_W}{s_W}\hat{Z}_\mu)\frac{\delta\Gamma}{\delta\hat{W}^\pm_\mu} \\
& \mp ie\frac{1}{2s_W}\hat{\phi}^\mp\Big(\frac{\delta\Gamma}{\delta\hat{H}} \pm i\frac{\delta\Gamma}{\delta\hat{\chi}}\Big) \pm ie\frac{1}{2s_W}(v + \hat{H} \pm i\hat{\chi})\frac{\delta\Gamma}{\delta\hat{\phi}^\pm} \\
& - ie\frac{1}{\sqrt{2}s_W}\sum_{(f_+,f_-)}\Big(\bar{f}_\pm \frac{1+\gamma_5}{2}\frac{\delta\Gamma}{\delta\bar{f}_\mp} + \frac{\delta\Gamma}{\delta f_\pm}\frac{1-\gamma_5}{2} f_\mp\Big),
\end{aligned} \tag{6}
$$

where $v_f = (I^3_{W,f} - 2s_W^2 Q_f)/(2s_W c_W)$ and $a_f = I^3_{W,f}/(2s_W c_W)$. In Eq. (6) $f_\pm$ denote the fermions with isospin $\pm 1/2$, and the sum in the last line runs over all isospin doublets. The electric unit charge is denoted by e as usual, and the Weinberg angle θ_W is fixed by the mass ratio,

$$
c_W = \cos\theta_W = \frac{M_W}{M_Z}, \qquad s_W = \sin\theta_W = \sqrt{1 - c_W^2}. \tag{7}
$$

By differentiating Eq. (6) with respect to background fields and setting the fields equal to zero, one obtains Ward identities for the vertex function that are precisely the Ward identities related to the classical Lagrangian. This is in contrast to the conventional formalism where, owing to the gauge-fixing procedure, explicit gauge invariance is lost, and Ward identities are obtained from invariance under BRS transformations. These Slavnov–Taylor identities have a more complicated structure and in general involve ghost contributions.

The BFM Ward identities are valid in all orders of perturbation theory and hold for arbitrary values of the quantum gauge parameter ξ_Q. They relate one-particle irreducible Green functions. In particular, the two-point functions do not contain tadpole contributions. These appear explicitly in the Ward identities.

For illustration and later use, we list some of the Ward identities. Concerning the notation and conventions for the vertex functions we follow Ref. 11 throughout. The two-point functions fulfill the following Ward identities:

$$
k^\mu \Gamma^{\hat{A}\hat{A}}_{\mu\nu}(k) = 0, \qquad k^\mu \Gamma^{\hat{A}\hat{Z}}_{\mu\nu}(k) = 0, \tag{8}
$$

$$
k^\mu \Gamma^{\hat{A}\hat{H}}_{\mu}(k) = 0, \qquad k^\mu \Gamma^{\hat{A}\hat{\chi}}_{\mu}(k) = 0, \tag{9}
$$

$$
k^\mu \Gamma^{\hat{Z}\hat{Z}}_{\mu\nu}(k) - iM_Z \Gamma^{\hat{\chi}\hat{Z}}_{\nu}(k) = 0, \tag{10}
$$

$$
k^\mu \Gamma^{\hat{Z}\hat{\chi}}_{\mu}(k) - iM_Z \Gamma^{\hat{\chi}\hat{\chi}}(k) + \frac{ie}{2s_W c_W}\Gamma^{\hat{H}}(0) = 0, \tag{11}
$$

$$k^\mu \Gamma^{\hat{W}^\pm \hat{W}^\mp}_{\mu\nu}(k) \mp M_{\mathrm{W}} \Gamma^{\hat{\phi}^\pm \hat{W}^\mp}_{\nu}(k) = 0, \tag{12}$$

$$k^\mu \Gamma^{\hat{W}^\pm \hat{\phi}^\mp}_{\mu}(k) \mp M_{\mathrm{W}} \Gamma^{\hat{\phi}^\pm \hat{\phi}^\mp}(k) \pm \frac{e}{2s_{\mathrm{W}}} \Gamma^{\hat{H}}(0) = 0. \tag{13}$$

The Ward identities for the gauge-boson–fermion vertices read

$$k^\mu \Gamma^{\hat{A}\bar{f}f}_{\mu}(k,\bar{p},p) = -eQ_f[\Gamma^{\bar{f}f}(\bar{p}) - \Gamma^{\bar{f}f}(-p)], \tag{14}$$

$$k^\mu \Gamma^{\hat{Z}\bar{f}f}_{\mu}(k,\bar{p},p) - iM_{\mathrm{Z}}\Gamma^{\hat{\chi}\bar{f}f}(k,\bar{p},p) = e[\Gamma^{\bar{f}f}(\bar{p})(v_f - a_f\gamma_5) - (v_f + a_f\gamma_5)\Gamma^{\bar{f}f}(-p)], \tag{15}$$

$$k^\mu \Gamma^{\hat{W}^\pm \bar{f}_\pm f_\mp}_{\mu}(k,\bar{p},p) \mp M_{\mathrm{W}}\Gamma^{\hat{\phi}^\pm \bar{f}_\pm f_\mp}(k,\bar{p},p) = \frac{e}{\sqrt{2}s_{\mathrm{W}}}[\Gamma^{\bar{f}_\pm f_\pm}(\bar{p})\omega_- - \omega_+\Gamma^{\bar{f}_\mp f_\mp}(-p)]. \tag{16}$$

The triple-gauge-boson vertices obey

$$k^\mu \Gamma^{\hat{A}\hat{W}^+\hat{W}^-}_{\mu\rho\sigma}(k,k_+,k_-) = e[\Gamma^{\hat{W}^+\hat{W}^-}_{\rho\sigma}(k_+) - \Gamma^{\hat{W}^+\hat{W}^-}_{\rho\sigma}(-k_-)], \tag{17}$$

$$\begin{aligned} k_+^\rho \Gamma^{\hat{A}\hat{W}^+\hat{W}^-}_{\mu\rho\sigma}(k,k_+,k_-) \; - \; & M_{\mathrm{W}}\Gamma^{\hat{A}\hat{\phi}^+\hat{W}^-}_{\mu\sigma}(k,k_+,k_-) = \\ & +e\left[\Gamma^{\hat{W}^+\hat{W}^-}_{\mu\sigma}(-k_-) - \Gamma^{\hat{A}\hat{A}}_{\mu\sigma}(k) + \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\Gamma^{\hat{A}\hat{Z}}_{\mu\sigma}(k)\right], \end{aligned} \tag{18}$$

$$\begin{aligned} k_-^\sigma \Gamma^{\hat{A}\hat{W}^+\hat{W}^-}_{\mu\rho\sigma}(k,k_+,k_-) \; + \; & M_{\mathrm{W}}\Gamma^{\hat{A}\hat{W}^+\hat{\phi}^-}_{\mu\rho}(k,k_+,k_-) = \\ & -e\left[\Gamma^{\hat{W}^-\hat{W}^+}_{\mu\rho}(-k_+) - \Gamma^{\hat{A}\hat{A}}_{\mu\rho}(k) + \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\Gamma^{\hat{A}\hat{Z}}_{\mu\rho}(k)\right], \end{aligned} \tag{19}$$

$$\begin{aligned} k^\mu \Gamma^{\hat{Z}\hat{W}^+\hat{W}^-}_{\mu\rho\sigma}(k,k_+,k_-) \; - \; & iM_{\mathrm{Z}}\Gamma^{\hat{\chi}\hat{W}^+\hat{W}^-}_{\rho\sigma}(k,k_+,k_-) = \\ & -e\frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}[\Gamma^{\hat{W}^+\hat{W}^-}_{\rho\sigma}(k_+) - \Gamma^{\hat{W}^+\hat{W}^-}_{\rho\sigma}(-k_-)], \end{aligned} \tag{20}$$

$$\begin{aligned} k_+^\rho \Gamma^{\hat{Z}\hat{W}^+\hat{W}^-}_{\mu\rho\sigma}(k,k_+,k_-) \; - \; & M_{\mathrm{W}}\Gamma^{\hat{Z}\hat{\phi}^+\hat{W}^-}_{\mu\sigma}(k,k_+,k_-) = \\ & -e\frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\left[\Gamma^{\hat{W}^+\hat{W}^-}_{\mu\sigma}(-k_-) - \Gamma^{\hat{Z}\hat{Z}}_{\mu\sigma}(k) + \frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}\Gamma^{\hat{Z}\hat{A}}_{\mu\sigma}(k)\right], \end{aligned} \tag{21}$$

$$\begin{aligned} k_-^\sigma \Gamma^{\hat{Z}\hat{W}^+\hat{W}^-}_{\mu\rho\sigma}(k,k_+,k_-) \; + \; & M_{\mathrm{W}}\Gamma^{\hat{Z}\hat{W}^+\hat{\phi}^-}_{\mu\rho}(k,k_+,k_-) = \\ & +e\frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\left[\Gamma^{\hat{W}^-\hat{W}^+}_{\mu\rho}(-k_+) - \Gamma^{\hat{Z}\hat{Z}}_{\mu\rho}(k) + \frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}\Gamma^{\hat{Z}\hat{A}}_{\mu\rho}(k)\right]. \end{aligned} \tag{22}$$

Note that the Ward identities involving only fermions and photons are exactly those of QED.

3. Renormalization of the Standard Model

3.1. Impact of Gauge Invariance on Renormalization

The BFM gauge invariance has important consequences for the structure of the renormalization constants necessary to render Green functions and S-matrix elements finite. The arguments which we give in the following are made explicit for the one-loop level.[c] It is easy, however, to extend them by induction to arbitrary orders in

[c] We implicitly assume the existence of an invariant regularization scheme.

perturbation theory. Because the renormalization of the fermionic sector is similar to the one in the conventional formalism, we leave it out[d].

We introduce the following renormalization constants for the parameters:

$$\begin{aligned}
e_0 &= Z_e e = (1+\delta Z_e)e, \\
M_{\mathrm{W},0}^2 &= M_{\mathrm{W}}^2 + \delta M_{\mathrm{W}}^2, \qquad M_{\mathrm{Z},0}^2 = M_{\mathrm{Z}}^2 + \delta M_{\mathrm{Z}}^2, \qquad M_{\mathrm{H},0}^2 = M_{\mathrm{H}}^2 + \delta M_{\mathrm{H}}^2, \\
t_0 &= t + \delta t.
\end{aligned} \tag{23}$$

The tadpole counterterm δt renormalizes the term in the Lagrangian linear in the Higgs field $\hat{H}$, which we denote by $t\hat{H}(x)$ with $t = v(\mu^2 - \lambda v^2/4)$. It corrects for the shift in the minimum of the Higgs potential due to radiative corrections. Choosing v as the correct vacuum expectation value of the Higgs field $\hat{\Phi}$ is equivalent to the vanishing of t. In principle, the renormalization constant δt is not necessary, and one could work with arbitrary or even without tadpole renormalization. In these cases, however, one would have to take into account explicit tadpole contributions.

Following the QCD treatment of Ref. 8, we introduce field renormalization only for the background fields

$$\begin{aligned}
\hat{W}_0^{\pm} &= Z_{\hat{W}}^{1/2}\hat{W}^{\pm} = (1+\frac{1}{2}\delta Z_{\hat{W}})\hat{W}^{\pm}, \\
\begin{pmatrix} \hat{Z}_0 \\ \hat{A}_0 \end{pmatrix} &= \begin{pmatrix} Z_{\hat{Z}\hat{Z}}^{1/2} & Z_{\hat{Z}\hat{A}}^{1/2} \\ Z_{\hat{A}\hat{Z}}^{1/2} & Z_{\hat{A}\hat{A}}^{1/2} \end{pmatrix} \begin{pmatrix} \hat{Z} \\ \hat{A} \end{pmatrix} = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{\hat{Z}\hat{Z}} & \frac{1}{2}\delta Z_{\hat{Z}\hat{A}} \\ \frac{1}{2}\delta Z_{\hat{A}\hat{Z}} & 1+\frac{1}{2}\delta Z_{\hat{A}\hat{A}} \end{pmatrix} \begin{pmatrix} \hat{Z} \\ \hat{A} \end{pmatrix}, \\
\hat{H}_0 &= Z_{\hat{H}}^{1/2}\hat{H} = (1+\frac{1}{2}\delta Z_{\hat{H}})\hat{H}, \\
\hat{\chi}_0 &= Z_{\hat{\chi}}^{1/2}\hat{\chi} = (1+\frac{1}{2}\delta Z_{\hat{\chi}})\hat{\chi}, \\
\hat{\phi}_0^{\pm} &= Z_{\hat{\phi}}^{1/2}\hat{\phi}^{\pm} = (1+\frac{1}{2}\delta Z_{\hat{\phi}})\hat{\phi}^{\pm}.
\end{aligned} \tag{24}$$

In order to preserve the background-field gauge invariance, the renormalized effective action has to be invariant under background-field gauge transformations. This restricts the possible counterterms and relates the renormalization constants introduced above. These relations can be derived from the requirement that the renormalized vertex functions fulfill Ward identities of the same form as the unrenormalized ones. As a consequence, also the counterterms have to fulfill these Ward identities. An analysis of the Ward identities yields [11]:

$$\begin{aligned}
\delta Z_{\hat{A}\hat{A}} &= -2\delta Z_e, \qquad \delta Z_{\hat{Z}\hat{A}} = 0, \qquad \delta Z_{\hat{A}\hat{Z}} = 2\frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2}, \\
\delta Z_{\hat{Z}\hat{Z}} &= -2\delta Z_e - \frac{c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2}, \qquad \delta Z_{\hat{W}} = -2\delta Z_e - \frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2},
\end{aligned}$$

[d]It is included in Ref. 11.

$$\delta Z_{\hat{H}} = \delta Z_{\hat{\chi}} = \delta Z_{\hat{\phi}} = -2\delta Z_e - \frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2} + \frac{\delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2}, \tag{25}$$

where

$$\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2} = \frac{\delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2} - \frac{\delta M_{\mathrm{Z}}^2}{M_{\mathrm{Z}}^2}. \tag{26}$$

The relations Eq. (25) express the field renormalization constants of all gauge bosons and scalars completely in terms of the renormalization constants of the electric charge and the particle masses. With this set of renormalization constants all background-field vertex functions become finite[e]. This is evident since the divergences of the vertex functions are subject to the same restriction as the counterterms. In Ref. 11 it has been verified explicitly at one-loop order that a renormalization based on the on-shell definition of all parameters can consistently be used in the BFM. It renders all vertex functions finite while respecting the full gauge symmetry of the BFM.

As the field renormalization constants are fixed by Eq. (25), the propagators in general acquire residues being different from unity but finite. This is similar to the minimal on-shell scheme of the conventional formalism[21] and has to be corrected in the S-matrix elements by UV-finite wave-function renormalization constants. However, just as in QED, the on-shell definition of the electric charge together with gauge invariance automatically fixes the residue of the photon propagator to unity.

As a consequence of the relations between the renormalization constants, the counterterm vertices of the background fields have a much simpler structure than the ones in the conventional formalism (see e.g. Ref. 19). In fact, all vertices originating from a separately gauge-invariant term in the Lagrangian acquire the same renormalization constants. The explicit form of the counterterm vertices at one-loop order has been given in Ref. 11.

3.2. Gauge-Parameter Independence of Counterterms and Running Couplings

If the renormalized parameters are identified with the physical electron charge and the physical particle masses, they are manifestly gauge-independent. Moreover, the original bare parameters in the Lagrangian are obviously gauge-independent, as they represent free parameters of the theory. The same is true for the bare charge and the bare weak mixing angle as these are directly related to the free bare parameters. Consequently, the counterterms δZ_e and δc_{W}^2 for the gauge couplings are gauge-independent. The relations Eq. (25) therefore imply that the field renormalizations of all gauge-boson fields are gauge-independent. This is in contrast to the

[e] Beyond one-loop order one needs in addition a renormalization of the quantum gauge parameters [8]. At one-loop level these counterterms do not enter the background-field vertex functions because ξ_Q does not appear in pure background-field vertices. Clearly, the renormalization of gauge parameters is irrelevant for gauge-independent quantities such as S-matrix elements at any order.

conventional formalism where the field renormalizations in the on-shell scheme are gauge-dependent.

In contrast to δZ_e and δc_{W}^2 the mass counterterms are not gauge-independent. The bare masses depend on the bare vacuum expectation value v_0 of the Higgs field, which is not a free parameter of the theory. Whereas the renormalized value $v = 2s_{\mathrm{W}}M_{\mathrm{W}}/e$ is gauge-independent, the bare quantity v_0 and the corresponding counterterm δv are not. As a consequence, the bare masses are gauge-dependent. Thus, the counterterms δM_{W}^2, δM_{Z}^2, δM_{H}^2, δm_f and δt are also gauge-dependent. The physical masses, however, are determined by the pole positions of the propagators, i.e. the zeros of $k^2 - M^2 - \delta M^2 + C\delta t/M_{\mathrm{H}}^2 + \Sigma(k^2) + CT^{\hat{H}}/M_{\mathrm{H}}^2$, where C denotes the coupling of the fields to the Higgs field and $\Sigma(k^2)$ the relevant self-energy. The linear combination $\delta M^2 - C\delta t/M_{\mathrm{H}}^2$ of the mass and tadpole counterterm, however, is independent of δv and thus gauge-independent[f].

Just as in QED, one can define running couplings in the BFM for the SM via naïve Dyson summation of self-energies as follows:

$$
\begin{aligned}
e^2(q^2) &= \frac{e_0^2}{1 + \mathrm{Re}\,\Pi_0^{\hat{A}\hat{A}}(q^2)} = \frac{e^2}{1 + \mathrm{Re}\,\Pi^{\hat{A}\hat{A}}(q^2)}, \\
g_2^2(q^2) &= \frac{g_{2,0}^2}{1 + \mathrm{Re}\,\Pi_0^{\hat{W}\hat{W}}(q^2)} = \frac{g_2^2}{1 + \mathrm{Re}\,\Pi^{\hat{W}\hat{W}}(q^2)},
\end{aligned}
\tag{27}
$$

where

$$
g_{2,0} = \frac{e_0}{s_{\mathrm{W},0}} \quad \text{and} \quad g_2 = \frac{e}{s_{\mathrm{W}}}, \tag{28}
$$

and the subscript "0" denotes bare quantities. The quantities $\Pi^{\hat{V}\hat{V}'}$ are related to the transverse parts of the gauge-boson self-energies as follows:

$$
\Pi^{\hat{V}\hat{V}'}(q^2) = \frac{\Sigma_{\mathrm{T}}^{\hat{V}\hat{V}'}(q^2) - \Sigma_{\mathrm{T}}^{\hat{V}\hat{V}'}(0)}{q^2}. \tag{29}
$$

The relations Eq. (25) give rise to a number of nice properties of the running couplings. As indicated in Eq. (27), the renormalization constants cancel. Consequently, the running couplings are finite without renormalization and thus independent of the renormalization scheme (as long as it respects BFM gauge invariance). Their asymptotic behavior is gauge-independent and governed by the renormalization group. In particular, the coefficients of the leading logarithms in the self-energies are equal to the ones appearing in the β-functions associated with the running couplings. All these properties are completely analogous to those of the running coupling in QED; they follow in the same way from the relations Eq. (25) as in QED from $Z_e = Z_{\hat{A}\hat{A}}^{-1/2}$.

As mentioned above, the asymptotic behavior of $e^2(q^2)$ and $g_2^2(q^2)$ is independent of the quantum gauge parameter. The running couplings coincide in this region with those defined in Refs. 1,2,4. For finite values of q^2, however, there are differences[g],

[f] Note that the mass counterterms become gauge-independent if one chooses $\delta t = 0$.

[g] Those differences also exist between the different formulations of the previous treatments[1,2,4].

and the couplings Eq. (27) depend on ξ_Q. This indicates that the mentioned desirable theoretical properties do not single out any specific definition of the running couplings. Instead, any definition of running couplings via Dyson summation of self-energies that take into account mass effects is not unique but a matter of convention. This arbitrariness is made transparent in the BFM and has to be taken into account in treatments based on running couplings.

4. Non-linear Realization of the scalar Sector

In the previous sections the scalar $\mathrm{SU(2)_W}$ doublet $\hat{\Phi}$ was represented in the usual linear way, as defined in Eq. (2). It is interesting to inspect also the non-linear realization of the scalar sector specified by [22,23]

$$\hat{\Phi} = \frac{1}{\sqrt{2}}(v + \hat{H})\hat{U}, \tag{30}$$

where the Goldstone fields $\hat{\phi}^a$ form the unitary matrix $\hat{U}$. A convenient representation for $\hat{U}$ is for instance given by

$$\hat{U} = \exp\left(\frac{i}{v}\hat{\phi}^a\sigma^a\right). \tag{31}$$

The $\hat{\phi}^a$ are related to the charge eigenstates $\hat{\phi}^\pm$, $\hat{\chi}$ as follows

$$\hat{\phi}^\pm = \frac{1}{\sqrt{2}}\left(\hat{\phi}^2 \pm i\hat{\phi}^1\right), \qquad \hat{\chi} = -\hat{\phi}^3. \tag{32}$$

The (physical) Higgs field $\hat{H}$ is a $\mathrm{SU(2)_W}$ singlet unlike in the linear parametrization of Eq. (2). The (non-polynomial) Higgs part of the Lagrangian reads

$$\mathcal{L}_H = \frac{1}{2}\,\mathrm{tr}\left\{(\hat{D}_\mu\hat{\Phi})^\dagger(\hat{D}^\mu\hat{\Phi})\right\} + \frac{1}{2}\mu^2\,\mathrm{tr}\left\{\hat{\Phi}^\dagger\hat{\Phi}\right\} - \frac{1}{16}\lambda\left(\mathrm{tr}\left\{\hat{\Phi}^\dagger\hat{\Phi}\right\}\right)^2, \tag{33}$$

where $\hat{D}_\mu$ denotes the covariant derivative of $\hat{\Phi}$ in matrix notation

$$\hat{D}^\mu\hat{\Phi} = \partial_\mu\hat{\Phi} - ig_2\hat{W}^a_\mu\frac{\sigma^a}{2}\hat{\Phi} - ig_1\hat{\Phi}\hat{B}_\mu\frac{\sigma^3}{2}. \tag{34}$$

One of the most interesting features of the non-linear realization Eq. (30) is that the scalar self interaction in Eq. (33) is independent of the unphysical Goldstone fields $\hat{\phi}^a$ owing to the unitarity of $\hat{U}$. The linear and non-linear realizations of the scalar sector turn out to be physically equivalent [22], as the Jacobian of the corresponding field transformation yields only a contribution to the Lagrangian proportional to $\delta^{(D)}(0)$, which cancels extra quartic UV divergences occurring in loop diagrams but vanishes anyhow in dimensional regularization.

In the BFM the fields $\hat{H}$ and $\hat{\phi}^a$ are split into background and quantum fields as follows [13]

$$\hat{H} \to \hat{H} + H, \qquad \hat{U} \to \hat{U}U. \tag{35}$$

Note that in order to preserve background gauge invariance the matrix $\hat{U}$ is split multiplicatively, i.e. the $\hat{\phi}^a$ are split in a non-linear way. The corresponding R_ξ-gauge-fixing term for the quantum fields reads [13]

$$\begin{aligned}\mathcal{L}_{\mathrm{GF}} = &-\frac{1}{4\xi_Q}\,\mathrm{tr}\left\{\left(\partial^\mu W^a_\mu \sigma^a + g_2 \varepsilon^{abc}\hat{W}^a_\mu W^{\mu,b}\sigma^c + \xi_Q \frac{g_2 v}{2}\hat{U}\phi^a\sigma^a\hat{U}^\dagger\right)^2\right\}\\ &-\frac{1}{2\xi_Q}\left(\partial^\mu B_\mu + \xi_Q\frac{g_1 v}{2}\phi^3\right)^2,\end{aligned} \tag{36}$$

and the Faddeev–Popov ghost Lagrangian $\mathcal{L}_{\mathrm{FP}}$ is constructed as usual. Since $\mathcal{L}_{\mathrm{GF}}$ does not involve H and $\hat{H}$, the physical Higgs field does not couple to the Faddeev–Popov ghost fields.

Owing to the gauge invariance of the background Higgs field $\hat{H}$, vertex functions involving only $\hat{H}$ fields are independent of the gauge parameter ξ_Q. We have explicitly checked this for the case of the tadpole $\Gamma^{\hat{H}} = iT^{\hat{H}}$ and the Higgs two-point function $\Gamma^{\hat{H}\hat{H}}(q) = i(q^2 - M_{\mathrm{H}}^2) + i\Sigma^{\hat{H}\hat{H}}(q^2)$. Hence, the tadpole counterterm $\delta t = -T^{\hat{H}}$ and the Higgs-boson mass counterterm $\delta M_{\mathrm{H}}^2 = \mathrm{Re}\left(\Sigma^{\hat{H}\hat{H}}(M_{\mathrm{H}}^2)\right)$ are gauge-independent in contrast to the corresponding quantities in the linear parametrization. Moreover, the gauge independence of δt implies the same for the gauge-boson mass counterterms δM_{W}^2 and δM_{Z}^2 (and for the fermion-mass counterterms) because of gauge independence of propagator poles.

Carrying out the field renormalization in a way respecting background-field gauge invariance, one finds the same relations, Eq. (25), for the field renormalization constants as in the linear scalar realization except for the one of $\delta Z_{\hat{H}}$. There is no constraint on $\delta Z_{\hat{H}}$ following from gauge invariance.

In this context we mention that the non-polynomial scalar self interactions in Eq. (33) lead to a Higgs self-energy $\Sigma^{\hat{H}\hat{H}}(q^2)$ which off-shell remains UV-divergent even after Higgs-field and Higgs-mass renormalization. This is due to the presence of UV-divergent terms proportional to q^4. Of course, in S-matrix elements these spurious divergences always cancel against their counterparts in other vertex functions since the complete theory is renormalizable.

Disregarding the physical Higgs field in the non-linear realization Eq. (30), the SM reduces to the so-called gauged non-linear σ-model (GNLSM) [24]. The GNLSM is non-renormalizable but still a $\mathrm{SU(2)_W \times U(1)_Y}$ gauge theory. The BFM effective action of the GNLSM is gauge-invariant, and the corresponding vertex functions obey simple Ward identities. However, the structure of these Ward identities is different from the one in the SM described in the previous sections, although they can be derived analogously. This is due to the non-linearity in the scalar sector, which renders also

gauge transformations of the background Goldstone fields non-linear,

$$\delta\hat{\phi}^a = M_{\mathrm{W}}\delta\hat{\theta}^a + M_{\mathrm{W}}\frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}\delta\hat{\theta}^{\mathrm{Y}}\delta^{a3} - \frac{e}{2s_{\mathrm{W}}}\varepsilon^{abc}\delta\hat{\theta}^b\hat{\phi}^c + \frac{e}{2c_{\mathrm{W}}}\varepsilon^{a3c}\delta\hat{\theta}^{\mathrm{Y}}\hat{\phi}^c + \mathcal{O}(\hat{\phi}^2), \qquad (37)$$

as can be easily inferred from the detailed presentation of Ref. 13. Consequently, a Ward identity for an n-point function in general involves vertex functions with less external lines down to self-energies. Since H and $\hat{H}$ represent $\mathrm{SU}(2)_{\mathrm{W}} \times \mathrm{U}(1)_{\mathrm{Y}}$ singlets, the Ward identities of the GNLSM are valid in the SM with the non-linear scalar realization of Eq. (30), too. The remaining Ward identities in the SM with non-linear scalar sector, which involve $\hat{H}$ vertex functions, are obtained from the ones of the GNLSM simply by taking further functional derivatives with respect to $\hat{H}$, or diagrammatically by adding further $\hat{H}$ legs to each occurring vertex function. In particular, tadpole contributions can never occur in the Ward identities. In Eq. (37) the constant terms and the ones linear in the $\hat{\phi}^a$ coincide with the corresponding result for the linear realization of the scalar sector [see Eq. (21) of Ref. 11]. Therefore, Ward identities involving at most one Goldstone field but no Higgs field in each occurring vertex function coincide within the linear and non-linear scalar realizations. In particular, this is the case for all Ward identities given in section 2 except for Eqs. (11) and (13), which are modified in the non-linear scalar realization of the SM and the GNLSM to

$$k^\mu\Gamma_\mu^{\hat{Z}\hat{\chi}}(k) - iM_{\mathrm{Z}}\Gamma^{\hat{\chi}\hat{\chi}}(k) = 0, \qquad (38)$$

$$k^\mu\Gamma_\mu^{\hat{W}^\pm\hat{\phi}^\mp}(k) \mp M_{\mathrm{W}}\Gamma^{\hat{\phi}^\pm\hat{\phi}^\mp}(k) = 0, \qquad (39)$$

where no tadpole contributions occur.

5. Gauge Invariance and gauge-parameter-independent Formulations of Green Functions

In this section we discuss the relation between gauge invariance and gauge-parameter-independent formulations at the level of Green functions. One should be aware in this context that formally one can obtain a gauge-parameter-independent quantity in a totally trivial way, namely by putting the gauge parameters to a specific value, e.g. $\xi_i = 1$. A "trivial" gauge-parameter independence of this kind obviously is not related to any symmetry properties of the theory.

On the other hand, as mentioned in the introduction, the rearrangement of parts between different vertex functions in the conventional formalism of the SM according to the prescription of the pinch technique (PT) has led to new "vertex functions" that are gauge-parameter-independent and coincide with the corresponding vertex functions in the BFM for $\xi_Q = 1$. The PT "vertex functions" were found to fulfill the same Ward identities which within the BFM are a direct consequence of gauge invariance.

The origin of non-trivial symmetry relations in this case stems from the fact that the gauge parameters in the vertex functions are canceled while the lowest-order propagators connecting the PT "vertex functions" are still gauge-parameter-dependent. Obviously, this cannot be achieved by simply putting the gauge parameters in the conventionally defined vertex functions to a certain value. As the complete S-matrix element is independent of the gauge parameters, certain relations between the new "vertex functions" must exist that enforce the cancellation of the remaining gauge-parameter dependence.

It is important to note that the validity of non-trivial symmetry relations is not based on the actual gauge-parameter independence of the new "vertex functions", but — more generally — on the independence of the gauge parameters in the tree-level propagators from the gauge fixing within loop diagrams. This, however, is exactly the same situation as in the BFM. The vertex functions in the BFM depend on the quantum gauge parameter ξ_Q. This gauge dependence is completely unrelated to the gauge fixing entering the lowest-order propagators and giving rise to background gauge parameters[h] ξ_B^i. Thus, there is an analogy between the BFM and prescriptions for constructing gauge-parameter-independent "vertex functions" in the conventional formalism, as far as the cancellation of gauge-parameters associated with lowest-order quantities is concerned. In the BFM, however, the cancellation of background gauge parameters is enforced by the BFM Ward identities. Consequently, a possible (and particularly simple) choice for gauge-parameter-independent "vertex functions" constructed using the conventional formalism is one that respects the BFM Ward identities.

In order to illustrate this in some more detail, we treat as a simple example a four-fermion process $\mathrm{u}_1\bar{\mathrm{d}}_1 \to \mathrm{u}_2\bar{\mathrm{d}}_2$ at one-loop order, where u_i and d_i are up- and down-type fermions, respectively. For ease of notation we consider a charged current process, i.e. we do not include mixing effects between different gauge bosons. The complete one-loop contribution $\delta\mathcal{M}$ to the transition amplitude $\mathcal{M}$ can be written as

$$\begin{aligned}
\delta\mathcal{M} = {} & \left(\bar{d}_1\Gamma^{W^-\bar{d}_1u_1}_{\mu,(0)}u_1\right)\Delta^{W,\mu\alpha}\left(\Gamma^{W^+W^-}_{\alpha\beta,(1)} - i\Gamma^{W^+W^-H}_{\alpha\beta,(0)}\Gamma^H_{(1)}/M_\mathrm{H}^2\right)\Delta^{W,\beta\nu}\left(\bar{u}_2\Gamma^{W^+\bar{u}_2d_2}_{\nu,(0)}d_2\right) \\
& + \left(\bar{d}_1\Gamma^{W^-\bar{d}_1u_1}_{\mu,(0)}u_1\right)\Delta^{W,\mu\alpha}\left(\Gamma^{W^+\phi^-}_{\alpha,(1)} - i\Gamma^{W^+\phi^-H}_{\alpha,(0)}\Gamma^H_{(1)}/M_\mathrm{H}^2\right)\Delta^{\phi}\left(\bar{u}_2\Gamma^{\phi^+\bar{u}_2d_2}_{(0)}d_2\right) \\
& + \left(\bar{d}_1\Gamma^{\phi^-\bar{d}_1u_1}_{(0)}u_1\right)\Delta^{\phi}\left(\Gamma^{\phi^+W^-}_{\beta,(1)} - i\Gamma^{\phi^+W^-H}_{\beta,(0)}\Gamma^H_{(1)}/M_\mathrm{H}^2\right)\Delta^{W,\beta\nu}\left(\bar{u}_2\Gamma^{W^+\bar{u}_2d_2}_{\nu,(0)}d_2\right) \\
& + \left(\bar{d}_1\Gamma^{\phi^-\bar{d}_1u_1}_{(0)}u_1\right)\Delta^{\phi}\left(\Gamma^{\phi^+\phi^-}_{(1)} - i\Gamma^{\phi^+\phi^-H}_{(0)}\Gamma^H_{(1)}/M_\mathrm{H}^2\right)\Delta^{\phi}\left(\bar{u}_2\Gamma^{\phi^+\bar{u}_2d_2}_{(0)}d_2\right) \\
& + \left(\bar{d}_1\Gamma^{W^-\bar{d}_1u_1}_{\mu,(1)}u_1\right)\Delta^{W,\mu\nu}\left(\bar{u}_2\Gamma^{W^+\bar{u}_2d_2}_{\nu,(0)}d_2\right) + \left(\bar{d}_1\Gamma^{W^-\bar{d}_1u_1}_{\mu,(0)}u_1\right)\Delta^{W,\mu\nu}\left(\bar{u}_2\Gamma^{W^+\bar{u}_2d_2}_{\nu,(1)}d_2\right) \\
& + \left(\bar{d}_1\Gamma^{\phi^-\bar{d}_1u_1}_{(1)}u_1\right)\Delta^{\phi}\left(\bar{u}_2\Gamma^{\phi^+\bar{u}_2d_2}_{(0)}d_2\right) + \left(\bar{d}_1\Gamma^{\phi^-\bar{d}_1u_1}_{(0)}u_1\right)\Delta^{\phi}\left(\bar{u}_2\Gamma^{\phi^+\bar{u}_2d_2}_{(1)}d_2\right) \\
& + \bar{d}_1\bar{u}_2\Gamma^{\bar{d}_1u_1\bar{u}_2d_2}_{(1)}u_1d_2, \qquad (40)
\end{aligned}$$

[h]In this section we restrict ourselves to linear background gauge-fixing conditions. Note that the PT has only been formulated for linear gauge fixings.

where $\bar{d}_1, u_1, \bar{u}_2$, and d_2 denote the spinors of the external fermions. The subscripts "(0)" and "(1)" mark lowest-order and one-loop quantities, respectively. The terms in the first four lines are self-energy and tadpole contributions, the ones in the fifth and sixth line are vertex corrections, and the last line contains the one-loop box contribution. Since we are concerned with an S-matrix element, Eq. (40) is understood to contain *renormalized quantities* only. In particular, the wave function renormalizations of the external fermion lines are completely absorbed in the vertex corrections.

We use a linear R_ξ gauge for the lowest-order propagators Δ^ϕ and $\Delta^W_{\mu\nu}$, i.e.

$$\Delta^\phi(k) = \frac{i}{k^2 - \xi M_\mathrm{W}^2}, \quad \Delta^W_{\mu\nu}(k) = \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{M_\mathrm{W}^2}\right)\frac{i}{k^2 - M_\mathrm{W}^2} - \frac{k_\mu k_\nu}{M_\mathrm{W}^2}\Delta^\phi(k). \tag{41}$$

According to our discussion above, we assume that the gauge-parameter dependence of the one-particle irreducible contributions in Eq. (40) is not related to the one of the tree propagators, Eq. (41). This includes both the case of the BFM and of gauge-parameter-independent "vertex functions" constructed in the conventional formalism.

Since the box contribution is independent of the (background-type) gauge parameter ξ, the cancellation of ξ requires symmetry relations involving self-energy and vertex contributions. After inserting Eq. (41) into Eq. (40), the complete ξ dependence is contained in the term Δ^ϕ. Collecting these gauge-dependent parts yields two relations, namely one for the contributions proportional to $\left(\Delta^\phi\right)^2$ and one for the terms proportional to Δ^ϕ. Using the relations

$$\bar{d}_1 k^\mu \Gamma^{W^-\bar{d}_1 u_1}_{\mu,(0)} u_1 = M_\mathrm{W} \bar{d}_1 \Gamma^{\phi^-\bar{d}_1 u_1}_{(0)} u_1, \qquad \bar{u}_2 k^\nu \Gamma^{W^+\bar{u}_2 d_2}_{\nu,(0)} d_2 = M_\mathrm{W} \bar{u}_2 \Gamma^{\phi^+\bar{u}_2 d_2}_{(0)} d_2, \tag{42}$$

and the tensor structure of the two-point functions, we find

$$k^\alpha k^\beta \Gamma^{W^+W^-}_{\alpha\beta,(1)}(k) - 2M_\mathrm{W} k^\alpha \Gamma^{W^+\phi^-}_{\alpha,(1)}(k) + M_\mathrm{W}^2 \Gamma^{\phi^+\phi^-,(1)}(k) - \frac{eM_\mathrm{W}}{2s_\mathrm{W}}\Gamma^H_{(1)} = 0, \tag{43}$$

$$\begin{aligned}
&\left(\bar{d}_1 k^\mu \Gamma^{W^-\bar{d}_1 u_1}_{\mu,(0)} u_1\right) 2i \left[\frac{k^\alpha k^\beta}{k^2}\Gamma^{W^+W^-}_{\alpha\beta,(1)}(k) - \frac{M_\mathrm{W} k^\alpha}{k^2}\Gamma^{W^+\phi^-}_{\alpha,(1)}(k)\right] \left(\bar{u}_2 k^\nu \Gamma^{W^+\bar{u}_2 d_2}_{\nu,(0)} d_2\right) \\
&\qquad + \left(\bar{d}_1 k^\mu \Gamma^{W^-\bar{d}_1 u_1}_{\mu,(0)} u_1\right) \frac{ieM_\mathrm{W}}{s_\mathrm{W} M_\mathrm{H}^2}\Gamma^H_{(1)} \left(\bar{u}_2 k^\nu \Gamma^{W^+\bar{u}_2 d_2}_{\nu,(0)} d_2\right) \\
&\qquad + \left(\bar{d}_1 k^\mu \Gamma^{W^-\bar{d}_1 u_1}_{\mu,(0)} u_1\right) M_\mathrm{W}^2 \left[\bar{u}_2 k^\nu \Gamma^{W^+\bar{u}_2 d_2}_{\nu,(1)} d_2 - M_\mathrm{W} \bar{u}_2 \Gamma^{\phi^+\bar{u}_2 d_2}_{(1)} d_2\right] \\
&\qquad + M_\mathrm{W}^2 \left[\bar{d}_1 k^\mu \Gamma^{W^-\bar{d}_1 u_1}_{\mu,(1)} u_1 - M_\mathrm{W} \bar{d}_1 \Gamma^{\phi^-\bar{d}_1 u_1}_{(1)} u_1\right] \left(\bar{u}_2 k^\nu \Gamma^{W^+\bar{u}_2 d_2}_{\nu,(0)} d_2\right) = 0,
\end{aligned} \tag{44}$$

where k^μ represents the total incoming momentum of the initial state. Eq. (43) coincides with the (renormalized) Ward identity valid for the one-loop self-energies in the conventional formalism of the SM, while Eq. (44) involves both process-specific vertex contributions and process-independent self-energies and a tadpole term. Note that the self-energy and vertex contributions in Eq. (44) do not necessarily decouple.

In the particular case of the BFM with the renormalization procedure described in section 3, Eqs. (43) and (44) are obviously fulfilled. Eq. (43) is just the sum of the BFM Ward identities Eqs. (12) and (13). In Eq. (44) the four lines actually vanish separately. The first line is zero owing to the Ward identity Eq. (12), the second is absent since the tadpole is renormalized to zero, and the last two lines vanish owing to the Ward identities Eq. (16) and the on-shell conditions for the fermions.

In this context, it is interesting to add some remarks on the tadpole contributions. Of course, it is not necessary to renormalize the tadpole to zero as it is done in section 3. Instead, one can fix its renormalized value arbitrarily or one need not renormalize it at all. This leads to additional tadpole contributions in all mass counterterms, e.g.

$$\delta M_{\mathrm{W}}^2 = \mathrm{Re}\left(\Sigma_{0,\mathrm{T}}^{WW}(M_{\mathrm{W}}^2)\right) - \frac{eM_{\mathrm{W}}}{s_{\mathrm{W}}M_{\mathrm{H}}^2}T^H, \tag{45}$$

$$\delta m_f = \frac{1}{2}m_f\,\mathrm{Re}\Big[\Sigma_{0,\mathrm{L}}^{\bar{f}f}(m_f^2) + \Sigma_{0,\mathrm{R}}^{\bar{f}f}(m_f^2) + 2\Sigma_{0,\mathrm{S}}^{\bar{f}f}(m_f^2)\Big] - \frac{e\,m_f}{2s_{\mathrm{W}}M_{\mathrm{W}}M_{\mathrm{H}}^2}T^H, \tag{46}$$

where the unrenormalized self-energies Σ_0 are defined like in Ref. 11. The renormalized tadpole $T^H = T_0^H + \delta t$ consists of the unrenormalized tadpole contribution T_0^H and the tadpole counterterm δt. The tadpole terms in Eqs. (45) and (46) are canceled in $\delta\mathcal{M}$ by the tadpole contributions in Eq. (40). Consequently, in such a renormalization scheme the four lines in Eq. (44) do *not* decouple. Using the BFM with a finite renormalized tadpole, the situation is as follows: the first line of Eq. (44) is still zero owing to the Ward identity Eq. (12). However, the last two lines yield finite tadpole contributions upon inserting the identities Eqs. (16) and using the on-shell conditions for the fermions,

$$\mathrm{Re}\left\{\Gamma_{(1)}^{\bar{f}f}(-p) - \frac{i}{M_{\mathrm{H}}^2}\Gamma_{(0)}^{\bar{f}fH}(-p,p,0)\Gamma_{(1)}^{H}\right\}\Bigg|_{p^2=m_f^2} u(p) = 0. \tag{47}$$

The resulting terms cancel exactly against the tadpole contribution in Eq. (44).

The above investigation of the gauge-parameter dependence associated with the tree lines for the example of a (charged-current) four-fermion process shows, in particular, that the gauge independence of the corresponding S-matrix element does not require a decoupling of the conventional Ward identity Eq. (43) into the BFM counterparts Eqs. (12) and (13). This is in contrast to the statements made in Ref. 6 in the PT framework. There the decoupled Ward identities were derived under the *additional assumption* that the tree-like gauge-parameter dependence of self-energy contributions is canceled independently of the remaining vertex and tadpole contributions. In particular, care has to be taken with respect to the tadpole contributions. They cannot be simply included in the self-energies obeying the decoupled identities Eqs. (12) and (13), since they do not fulfill these identities by themselves. Finally, we emphasize that derivations starting from the gauge independence of the S-matrix

can only yield results for *renormalized* vertex functions since an "unrenormalized S-matrix" does not exist.

Even if gauge-parameter-independent "vertex functions" are constructed in such a way that they fulfill the BFM Ward identities, their definition is still not unique. One can always shift parts between the "vertex functions" that by themselves fulfill the Ward identities. This freedom naturally appears within the framework of the BFM as the freedom of choosing different values of the quantum gauge parameter ξ_Q. As has been stressed above, the BFM Ward identities and the desirable properties of the BFM vertex functions are a consequence of gauge invariance and hold for arbitrary values of ξ_Q.

When comparing the approach pursued e.g. in the PT with the BFM, one should keep in mind that the field-theoretical interpretation of the PT quantities being defined by a rearrangement of contributions between different vertex functions is rather obscure. Their process independence has not been proven in general, but only verified for specific examples (see in particular Ref. 5), and their construction beyond one-loop order is technically very complicated [25]. In contrast, the BFM vertex functions have a well-defined field-theoretical meaning and can be derived from the effective action in all orders of perturbation theory.

As we have seen above, in the conventional formalism the application of the PT is a special case of decoupling the gauge-parameter dependence of the vertex functions from the one in the tree propagators. Recently, however, the PT has also been applied within the framework of the BFM in order to eliminate the dependence of the BFM vertex functions on the quantum gauge parameter ξ_Q [26]. Since the background gauge parameters ξ_B appearing in the tree propagators of the BFM are not related to ξ_Q, an elimination of ξ_Q via a prescription like the PT can *not* be distinguished from trivially putting ξ_Q to any specific value. This can also be seen from the fact that application of the PT within the BFM does not lead to new relations between the BFM vertex functions. The apparent gauge-parameter independence has only been achieved on cost of the specific prescription used in the PT to eliminate the gauge parameter.

The comparison between PT and BFM has made transparent that, despite their gauge-parameter independence and several desirable properties, the PT "vertex functions" are not unique but to a large extent a matter of convention. This is evident, because any off-shell quantity cannot be directly related to an observable and thus cannot uniquely be fixed. It was already pointed out in Ref. 23 that off-shell quantities are ambiguous even if gauge invariance is imposed. This holds in particular for all off-shell form factors such as a neutrino electromagnetic moment or anomalous triple-gauge-boson couplings. While these quantities are well-defined where the single one-particle exchange approximation holds, like e.g. on the Z resonance, they are not directly observable in general and to a large extent ambiguous. The PT, like any other prescription, can only provide a more or less convenient definition for off-shell

quantities but cannot supply a physical meaning.

6. The S, T, and U Parameters in the Background-Field Method

As an illustration of the discussion given in the last section, we treat the S, T, and U parameters in the framework of the BFM. The S, T, and U parameters are defined as certain combinations of self-energies [15]. Originally, they were introduced in order to parametrize the effects of new physics that enters only via oblique (i.e. self-energy) corrections. They can be extracted from experiment by comparing the experimentally measured values $\mathcal{A}_i^{\rm exp}$ of a number of observables with their values predicted by the SM, $\mathcal{A}_i^{\rm SM}$, i.e.

$$\mathcal{A}_i^{\rm exp} = \mathcal{A}_i^{\rm SM} + f_i^{\rm NP}(S,T,U). \tag{48}$$

Here $\mathcal{A}_i^{\rm SM}$ contains the complete radiative corrections in the SM up to a given order, while $f_i^{\rm NP}(S,T,U)$ is a function of the parameters S, T, U and describes the contributions of new physics. The SM prediction $\mathcal{A}_i^{\rm SM}$ is evaluated for a reference value of $m_{\rm t}$ and $M_{\rm H}$. For most observables accessible by precision measurements the corrections caused by a variation of $m_{\rm t}$ and $M_{\rm H}$ can also be absorbed into the parameters S, T, and U.

The parameters S, T, and U obtained via Eq. (48) are gauge-invariant quantities. This follows from the fact that $\mathcal{A}_i^{\rm SM}$ contains a complete set of electroweak radiative corrections entering an S-matrix element and that the analysis has been restricted to those models of new physics where $f_i^{\rm NP}(S,T,U)$ accounts for the total contribution.

In Ref. 16, however, an extension of the S, T, and U parametrization to cases where these assumptions do not hold has been discussed. This includes effects of new physics that do not exclusively enter via oblique corrections but also via vertex and box contributions as for example anomalous triple-gauge-boson couplings. Furthermore, the authors of Ref. 16 also considered the case where the S, T, and U parameters are used *within* the SM, i.e. to parametrize not only new physics effects but also the SM fermionic and bosonic radiative corrections.

These extensions of the S, T, and U parameters appear to be questionable, since the parameters defined in this way are no longer directly related to observables. In particular, this poses severe problems of gauge invariance. It was pointed out in Ref. 16 that calculating the one-loop bosonic SM contributions to the S, T, and U parameters yields gauge-parameter-dependent results. It was further noted that for gauges with $\xi_W \neq c_{\rm W}^2\xi_Z + s_{\rm W}^2\xi_A$ the parameters T and U are even UV-divergent. The authors of Ref. 16 argued that these problems can be overcome by using the PT in order to eliminate the gauge-parameter dependence of the one-loop gauge-boson self-energies. By explicit calculation, the S, T, and U parameters obtained within the PT were also shown to be UV-finite.

In order to discuss the formulation of the S, T, and U parameters given in Ref. 16, we calculate the bosonic SM contributions to the S, T, and U parameters in the

framework of the BFM. To allow for an easy comparison, we adopt the same definition of S, T, and U as in Ref. 16, i.e.

$$\alpha S_0 = 4c_{\rm W}^2 s_{\rm W}^2 {\rm Re}\left\{-\Pi_0^{ZZ}(M_{\rm Z}^2) + \frac{s_{\rm W}^2 - c_{\rm W}^2}{c_{\rm W}s_{\rm W}}\Pi_0^{\gamma Z}(M_{\rm Z}^2) + \Pi_0^{\gamma\gamma}(M_{\rm Z}^2)\right\}, \tag{49}$$

$$\alpha T_0 = -\frac{\Sigma_{\rm T,0}^{WW}(0)}{M_{\rm W}^2} + \frac{\Sigma_{\rm T,0}^{ZZ}(0)}{M_{\rm Z}^2} - 2c_{\rm W}s_{\rm W}\frac{\Sigma_{\rm T,0}^{\gamma Z}(0)}{M_{\rm W}^2}, \tag{50}$$

$$\alpha U_0 = 4s_{\rm W}^2 {\rm Re}\left\{-\Pi_0^{WW}(M_{\rm W}^2) + c_{\rm W}^2\Pi_0^{ZZ}(M_{\rm Z}^2) - 2c_{\rm W}s_{\rm W}\Pi_0^{\gamma Z}(M_{\rm Z}^2) + s_{\rm W}^2\Pi_0^{\gamma\gamma}(M_{\rm Z}^2)\right\}, \tag{51}$$

where as usual $\alpha = e^2/(4\pi)$. We use the subscript "0" to indicate that S, T, and U are defined in terms of unrenormalized one-loop self-energies. Note that in our conventions $s_{\rm W}$ differs by a sign from the one used in Ref. 16. Furthermore, we use the on-shell definitions for e and $s_{\rm W}$, while in Ref. 16 the $\overline{\rm MS}$ parameters are used. This difference is irrelevant for the discussion in this section.

The bosonic contributions to S, T and U in the BFM formulation of the SM read

$$\begin{aligned}\alpha S_0^{\rm SM,BFM} = \frac{\alpha}{24\pi}\Big\{& 2c_{\rm W}^2 - 5 + h + 2c_{\rm W}^2\xi_Q - 2\log(c_{\rm W}^2) - 2\left[3 - (1+2c_{\rm W}^2)\xi_Q\right]\frac{\log(\xi_Q)}{1-\xi_Q} \\ & - 2(12 - 4h + h^2)F(M_{\rm Z}^2; M_{\rm H}, M_{\rm Z}) + 6(1 - 4c_{\rm W}^2\xi_Q)F(M_{\rm Z}^2; \sqrt{\xi_Q}M_{\rm W}, \sqrt{\xi_Q}M_{\rm W}) \\ & - 4\left[1 + 10c_{\rm W}^2 + c_{\rm W}^4 - 2c_{\rm W}^2(1+c_{\rm W}^2)\xi_Q + c_{\rm W}^4\xi_Q^2\right]F(M_{\rm Z}^2; M_{\rm W}, \sqrt{\xi_Q}M_{\rm W})\Big\},\end{aligned} \tag{52}$$

$$\begin{aligned}\alpha T_0^{\rm SM,BFM} = \frac{\alpha}{16\pi}\Big\{& \frac{1}{s_{\rm W}^2}(12 - 5\xi_Q) - \frac{3h^2}{c_{\rm W}^2(1-h)(c_{\rm W}^2 - h)}\log(h) \\ & + \frac{c_{\rm W}^2\log(c_{\rm W}^2)}{s_{\rm W}^4(c_{\rm W}^2 - h)(c_{\rm W}^2 - \xi_Q)(1 - c_{\rm W}^2\xi_Q)}\Big[3c_{\rm W}^2(3 - 2c_{\rm W}^2 + 3c_{\rm W}^4) - 3h(2 - c_{\rm W}^2 + 3c_{\rm W}^4) \\ & - \Big(9 - 12c_{\rm W}^2 + 23c_{\rm W}^4 + 9c_{\rm W}^6 - h(6/c_{\rm W}^2 - 9 + 20c_{\rm W}^2 + 12c_{\rm W}^4)\Big)\xi_Q \\ & + \Big(c_{\rm W}^2(5 + 6c_{\rm W}^2 + 11c_{\rm W}^4) - h(2 + 9c_{\rm W}^2 + 11c_{\rm W}^4)\Big)\xi_Q^2 - c_{\rm W}^2(2 + 3c_{\rm W}^2)(c_{\rm W}^2 - h)\xi_Q^3\Big] \\ & + \frac{3\left[3c_{\rm W}^2 - (3+2c_{\rm W}^2)\xi_Q + (1+2c_{\rm W}^2)\xi_Q^3 - c_{\rm W}^2\xi_Q^4\right]}{(c_{\rm W}^2 - \xi_Q)(1 - c_{\rm W}^2\xi_Q)}\frac{\log(\xi_Q)}{1-\xi_Q}\Big\},\end{aligned} \tag{53}$$

$$\begin{aligned}\alpha U_0^{\rm SM,BFM} = \frac{\alpha}{12\pi c_{\rm W}^2}\Big\{& -\frac{2}{c_{\rm W}^2} - \frac{39}{2} + \frac{171}{2}c_{\rm W}^2 - c_{\rm W}^4 + \frac{1}{2}hs_{\rm W}^2 + (4 + 12c_{\rm W}^2 - c_{\rm W}^4)\xi_Q \\ & + \frac{c_{\rm W}^4\log(c_{\rm W}^2)}{s_{\rm W}^2(c_{\rm W}^2 - \xi_Q)(1 - c_{\rm W}^2\xi_Q)}\Big[1 + 89c_{\rm W}^2 - 27c_{\rm W}^4 - (1/c_{\rm W}^2 + 107 - 41c_{\rm W}^2 + 44c_{\rm W}^4)\xi_Q \\ & + (13 + 53c_{\rm W}^2 - 33c_{\rm W}^4)\xi_Q^2 + 3c_{\rm W}^2(2 + 3c_{\rm W}^2)\xi_Q^3\Big] \\ & - \frac{s_{\rm W}^2}{(c_{\rm W}^2 - \xi_Q)(1 - c_{\rm W}^2\xi_Q)}\Big[1 + 5c_{\rm W}^2 + 27c_{\rm W}^4 - (1/c_{\rm W}^2 + 4 + 15c_{\rm W}^2 + 25c_{\rm W}^4)\xi_Q \\ & - (1/c_{\rm W}^2 + 12 - 6c_{\rm W}^2 + 13c_{\rm W}^4 - 2c_{\rm W}^6)\xi_Q^2\end{aligned}$$

$$+(1+22c_{\rm W}^2+16c_{\rm W}^4)\xi_Q^3-9c_{\rm W}^4\xi_Q^4\Big]\frac{\log(\xi_Q)}{1-\xi_Q}$$
$$-(12c_{\rm W}^2-4h+h^2/c_{\rm W}^2)F(M_{\rm W}^2;M_{\rm H},M_{\rm W})+c_{\rm W}^2(12-4h+h^2)F(M_{\rm Z}^2;M_{\rm H},M_{\rm Z})$$
$$+48c_{\rm W}^2s_{\rm W}^2F(M_{\rm W}^2;0,M_{\rm W})+\frac{s_{\rm W}^2}{c_{\rm W}^2}(1+5c_{\rm W}^2)(1-4c_{\rm W}^2\xi_Q)F(M_{\rm Z}^2;\sqrt{\xi_Q}M_{\rm W},\sqrt{\xi_Q}M_{\rm W})$$
$$-2s_{\rm W}^2(1+c_{\rm W}^2)\Big[1/c_{\rm W}^2+10+c_{\rm W}^2-2(1+c_{\rm W}^2)\xi_Q+c_{\rm W}^2\xi_Q^2\Big]F(M_{\rm Z}^2;M_{\rm W},\sqrt{\xi_Q}M_{\rm W})$$
$$-(1-4c_{\rm W}^2)(1/c_{\rm W}^2+20+12c_{\rm W}^2)\Big[F(M_{\rm W}^2;M_{\rm W},M_{\rm Z})-F(M_{\rm Z}^2;M_{\rm W},M_{\rm W})\Big]\Big\}, \quad (54)$$

where we have used the shorthand $h=M_{\rm H}^2/M_{\rm Z}^2$, and the quantum gauge parameter $\xi_Q=\xi_Q^W=\xi_Q^B$ has been kept as a free parameter. The UV-finite function $F(p^2;m_1,m_2)$ is defined as

$$F(p^2;m_1,m_2)=\mathrm{Re}\Big(B_0(p^2;m_1,m_2)-B_0(0;m_1,m_2)\Big)\,, \quad (55)$$

where $B_0(p^2;m_1,m_2)$ is the usual scalar one-loop two-point integral [19]. For completeness, we also give the difference between $S_0^{\rm SM,BFM}$, $T_0^{\rm SM,BFM}$, $U_0^{\rm SM,BFM}$ evaluated at $\xi_Q=1$ and the bosonic contributions to the S, T, and U parameters calculated in the 't Hooft–Feynman gauge (tHF) of the conventional formalism:

$$\begin{aligned}
\alpha S_0^{\rm SM,BFM}\Big|_{\xi_Q=1} &= \alpha S_0^{\rm SM,conv}\Big|_{\rm tHF}+\frac{2\alpha c_{\rm W}^2}{\pi}\mathrm{Re}\Big\{B_0(0,M_{\rm W},M_{\rm W})-B_0(M_{\rm Z}^2,M_{\rm W},M_{\rm W})\Big\}\,,\\
\alpha T_0^{\rm SM,BFM}\Big|_{\xi_Q=1} &= \alpha T_0^{\rm SM,conv}\Big|_{\rm tHF}+\frac{\alpha}{s_{\rm W}^2\pi}\Big\{B_0(0,M_{\rm W},M_{\rm W})-s_{\rm W}^2B_0(0,0,M_{\rm W})\\
&\qquad -c_{\rm W}^2B_0(0,M_{\rm W},M_{\rm Z})\Big\}\,,\\
\alpha U_0^{\rm SM,BFM}\Big|_{\xi_Q=1} &= \alpha U_0^{\rm SM,conv}\Big|_{\rm tHF}+\frac{4\alpha}{\pi}\mathrm{Re}\Big\{s_{\rm W}^2B_0(0,0,M_{\rm W})-c_{\rm W}^2B_0(0,M_{\rm W},M_{\rm W})\\
&\qquad +c_{\rm W}^2B_0(0,M_{\rm W},M_{\rm Z})-s_{\rm W}^2B_0(M_{\rm Z}^2,M_{\rm W},M_{\rm W})\Big\}\,. \quad (56)
\end{aligned}$$

This coincides with the result obtained within the PT given in Ref. 16.

As can be seen in Eqs. (52), (53) and (54), $S_0^{\rm SM,BFM}$, $T_0^{\rm SM,BFM}$, and $U_0^{\rm SM,BFM}$ are UV-finite for arbitrary values of ξ_Q. While within the PT the UV-finiteness of the parameters could only be inferred from explicit computation, in the BFM it is an obvious consequence of gauge invariance[i]. In order to show this, we consider the renormalized S, T and U parameters. The renormalization of $T_0^{\rm SM,BFM}$, for instance, yields

$$\begin{aligned}
\alpha T^{\rm SM,BFM} &= -\frac{\Sigma_{\rm T}^{\hat W\hat W}(0)}{M_{\rm W}^2}+\frac{\Sigma_{\rm T}^{\hat Z\hat Z}(0)}{M_{\rm Z}^2}\\
&= -\frac{\Sigma_{\rm T,0}^{\hat W\hat W}(0)}{M_{\rm W}^2}+\delta Z_{\hat W}+\frac{\delta M_{\rm W}^2}{M_{\rm W}^2}+\frac{\Sigma_{\rm T,0}^{\hat Z\hat Z}(0)}{M_{\rm Z}^2}-\delta Z_{\hat Z\hat Z}-\frac{\delta M_{\rm Z}^2}{M_{\rm Z}^2}, \quad (57)
\end{aligned}$$

[i] Note that in the BFM gauge invariance restricts the number of quantum gauge parameters to two, ξ_Q^W and ξ_Q^B. This automatically implies the identity $\xi_Q^W=c_{\rm W}^2\xi_Q^Z+s_{\rm W}^2\xi_Q^A$.

where we have used that in the BFM $\Sigma^{\hat{A}\hat{Z}}_{\mathrm{T},0}(0) = \Sigma^{\hat{A}\hat{Z}}_{\mathrm{T}}(0) = 0$ holds, which can be inferred from Eq. (8). However, from Eq. (25) we find

$$\delta Z_{\hat{W}} + \frac{\delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2} - \delta Z_{\hat{Z}\hat{Z}} - \frac{\delta M_{\mathrm{Z}}^2}{M_{\mathrm{Z}}^2} = 0, \tag{58}$$

and therefore

$$\alpha T^{\mathrm{SM,BFM}} = \alpha T_0^{\mathrm{SM,BFM}} = -\frac{\Sigma_{\mathrm{T}}^{\hat{W}\hat{W}}(0)}{M_{\mathrm{W}}^2} + \frac{\Sigma_{\mathrm{T}}^{\hat{Z}\hat{Z}}(0)}{M_{\mathrm{Z}}^2}. \tag{59}$$

Since $T_0^{\mathrm{SM,BFM}}$ and $T^{\mathrm{SM,BFM}}$ are identical, the unrenormalized parameter $T_0^{\mathrm{SM,BFM}}$ is manifestly UV-finite. Similarly one derives

$$\alpha S^{\mathrm{SM,BFM}} = \alpha S_0^{\mathrm{SM,BFM}}, \quad \alpha U^{\mathrm{SM,BFM}} = \alpha U_0^{\mathrm{SM,BFM}}. \tag{60}$$

For fermionic contributions, the combination of self-energies appearing in Eq. (59) is just the one-loop correction to the ρ parameter. While the bosonic contributions to this combination of self-energies are divergent in the conventional formalism of the SM, they are finite within the BFM.

Recalling the discussion of the previous section, it should now be obvious that the definition of the S, T, and U parameters based on the PT is not distinguished, neither through its UV-finiteness nor through its apparent gauge-parameter independence. This ambiguity reflects the fact that a parametrization of the SM bosonic contributions in terms of S, T, and U cannot directly be compared to experimentally measured quantities. Moreover, there is a priori no reason why the S, T, and U parameters defined within the PT should include the dominant part of the bosonic contributions to electroweak observables. In fact, comparing for the bosonic contributions the complete one-loop result of the ρ parameter stated in Ref. 27 with the PT value of αT [16], one finds that the process-specific bosonic contributions that are not included in the PT definition of αT give by far the most important contribution. The bosonic PT result even has a sign different from the complete bosonic one-loop contribution to the ρ parameter.[j] Furthermore, from the analysis of LEP1 observables and muon decay carried out in Ref. 28 it can directly be seen that the (universal) bosonic corrections associated with the PT gauge-boson self-energies in general do *not* represent the dominant bosonic effects.

In summary, while well established for the treatment of new physics contributions entering solely via vacuum polarization effects, the framework of the S, T, and U parameters appears not to be favorable for an incorporation of SM bosonic corrections or of new physics effects going beyond oblique corrections. As we have seen, their definition becomes ambiguous in these cases. In order to study the complete SM contributions, it seems to be more appropriate to directly inspect observables or

[j] We have assumed an electron target and varied the Higgs mass between 50 and 1000 GeV.

(process-specific) effective parameters closely related to measurable quantities. For LEP1 physics such parametrizations were e.g. proposed in Ref. 29 and Refs. 30,28.

7. Conclusion

Quantizing a gauge theory within the background-field method (BFM) yields a manifestly gauge-invariant effective action for the underlying model. The application of this method to the electroweak Standard Model has been reviewed and further investigated. We have derived consequences of the simple Ward identities that follow directly from gauge invariance of the effective action. In particular, we have discussed the impact of BFM gauge invariance on renormalization. Moreover, we have considered the generalization of the BFM to the non-linear realization of the scalar sector of the Standard Model.

The interplay between gauge-parameter independence of the S-matrix and Ward identities relating vertex functions has been further explored. We have shown that any formalism that decouples the gauge-parameter dependence of the vertex functions from the one of the tree lines leads to symmetry constraints for the corresponding "vertex functions". These quantities are, however, not uniquely determined by this requirement, but it is possible to shift parts between "vertex functions" that by themselves obey the constraints. This fact signals the ambiguity which within the BFM is naturally made transparent by the dependence of the vertex functions on the quantum gauge parameter.

Although approaches based on a redistribution of parts between different Green functions may yield "vertex functions" that coincide with the corresponding quantities in the BFM, from a conceptual point of view these methods differ considerably. In addition to being technically rather complicated, approaches like the pinch technique suffer from severe theoretical shortcomings. In particular, the field-theoretical meaning of objects constructed by redistributions is not clear. In contrast, the BFM vertex functions have a well-defined field-theoretical interpretation and are derived from an effective action in all orders of perturbation theory.

The application of a gauge-parameter elimination procedure within the BFM degenerates to a trivial selection of a particular value for the quantum gauge parameter and thus to a mere convention.

Since off-shell quantities such as Green functions are not directly related to observables, they cannot be fixed on physical grounds. Therefore, any prescription that fixes these quantities can only be a more or less convenient definition but cannot be unique. We have illustrated this fact by calculating and discussing the (gauge-dependent) standard contributions to the S, T, and U parameters.

Note added

We would like to comment on remarks made in Ref. 31 concerning the connection between background-field method and pinch technique. There, Ref. 10 was cited in the context of the "erroneous impression" and the "naive expectation" that "Green's functions calculated within the background-field method should be completely gauge-invariant, and identical to the corresponding pinch-technique Green's functions". Furthermore, with respect to the gauge-parameter dependence of the background-field vertex functions, it was stated in Ref. 31 that "there" (Ref. 10) "was an attempt to assign a physical significance to this dependence". *None* of these statements has been made in Ref. 10, where all statements and conclusions are based on facts but not on the (irrelevant) "initial expectations" mentioned in Ref. 31. Note that one of our conclusions was that the gauge-parameter dependence in the BFM signals the fact that it is *not* possible to assign a physical significance to off-shell Green functions.

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